# $T/B$ scaling of magnetization in the mixed valent compound $\beta$-YbAlB$_4$

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**Abstract.** Here we provide the first clear evidence of Fermi-liquid breakdown in an intermediate valence system. We employ high precision magnetization measurements of the valence fluctuating superconductor $\beta$-YbAlB$_4$ to probe the quantum critical free energy down to temperatures far below the characteristic energy scale of the valence fluctuations. The observed $T/B$ scaling in the magnetization over three decades not only indicates unconventional quantum criticality, but places an upper bound on the critical magnetic field $|B_c| < 0.2$ mT, a value comparable with the Earth's magnetic field and six orders of magnitude smaller than the valence fluctuation scale. This tiny value of the upper bound on $B_c$, well inside the superconducting dome, raises the fascinating possibility that valence fluctuating $\beta$-YbAlB$_4$ is intrinsically quantum critical, without tuning the magnetic field, pressure, or composition: the first known example of such a phenomenon in a metal.

## 1. Introduction

The breakdown of Fermi liquid behavior in metals observed near a magnetic quantum critical point, challenges our current understanding of strongly correlated electrons. Archetypical quantum critical (QC) materials can be found in the heavy-fermion intermetallics, which are usually described as a Kondo lattice with a competition between local moment magnetism and conduction electron screening of the local moments (the Kondo effect) [1, 2, 3]. To date, all QC materials of this kind are known to have an almost integral valence which stabilizes the local moments considered essential for the criticality. By contrast, departures from integral valence associated with valence fluctuations are thought to promote screening of local moments, suppressing critical phenomena.

On the other hand, recent studies by our group revealed the first example of an Yb-based heavy fermion superconductivity in the new compound $\beta$-YbAlB$_4$ ($T_c = 80$ mK) [4, 5]. Pronounced non-Fermi liquid behavior above $T_c$ and its magnetic field dependence indicate that the system is a rare example of a pure metal that displays quantum criticality at ambient pressure and close to zero magnetic field [4]. Furthermore, recent hard x-ray photoemission spectroscopy measurements revealed intermediate valence of Yb$^{+2.75}$ providing the first unique example of
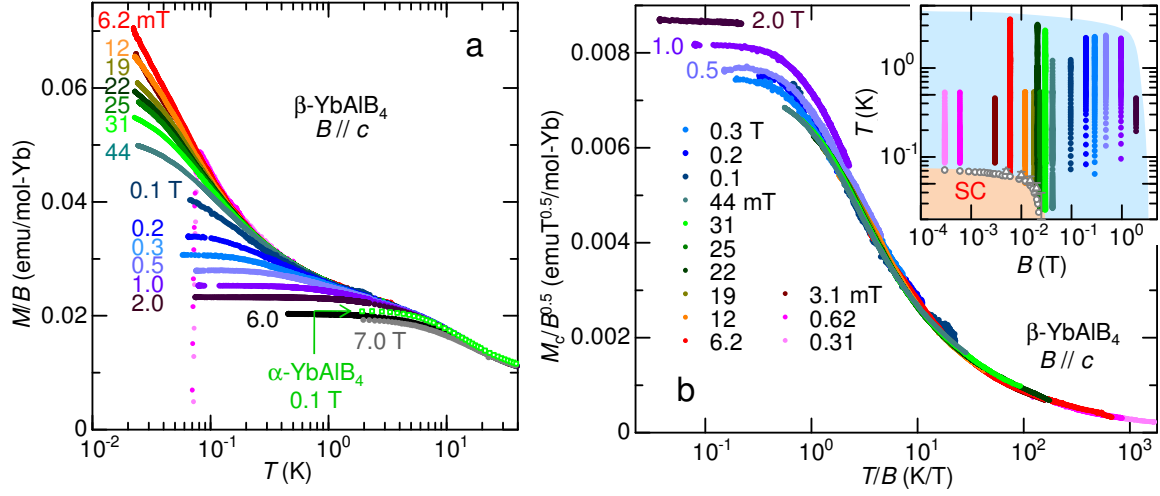
**Figure 1.** (a) Temperature dependence of the magnetic susceptibility $M/B$ of both $\beta$- (solid circles) and $\alpha$-YbAlB$_4$ (open squares) under the field $B \parallel c$. (b) Scaling plot of $M_c/B^{0.5}$ vs. $T/B$ for $\beta$-YbAlB$_4$. $M_c$ is the critical contribution to the magnetization: $M_c = M - M_0$ (see text). Inset shows the $B$-$T$ range where the scaling applies (solid circles in the blue shaded region) and the superconducting (SC) phase boundary (open circles and triangles [5, 8]) of $\beta$-YbAlB$_4$.

quantum criticality in a mixed valent system [6]. Interestingly, the system is governed by two distinct energy scales: a high-energy scale, $T_0 \sim 200$ K characterizing valence fluctuations, and a low-energy scale, $T^* \sim 8$ K characterizing the emergence of the Kondo lattice behavior [7].

Here we present clear experimental evidence that indicates the zero-field quantum criticality in $\beta$-YbAlB$_4$ [7]. We employed high resolution magnetization measurements and found a scaling equation $-dM/dT = B^{-1/2}f(T/B)$, which holds over wide ranges of magnetic fields (0.3 mT $< B <$ 2 T) and temperatures (20 mK $< T <$ 3 K). This $T/B$ scaling strongly suggests $B_c = 0$ within the experimental resolution of $\sim 0.2$ mT, which is comparable to the Earthfs magnetic field. This indicates a possibility of quantum critical phase that extends over a finite pressure range. The experimental details of this work have been discussed already in [7, 8].

## 2. Results and discussion

The divergent magnetic susceptibility $\chi$ at $T \to 0$ is one of the most remarkable feature which characterize the QC in $\beta$-YbAlB$_4$. Figure 1 shows the $T$ dependence of the $c$-axis susceptibility $\chi = M/B$ in the wide range of $T$ and $B$ spanning four orders of magnitude. Below $T \sim 3$ K, the results show a systematic evolution from a non-Fermi liquid (NFL) metal with divergent susceptibility at zero field to a Fermi liquid (FL) with finite susceptibility in a field.

Divergent behavior in NFL region is well fitted by a sum of the QC contribution $M_c/B \propto T^{-0.5}$ and $T$-independent constant term $\chi_0(= M_0/B) = 0.017$ (emu/mol). The latter constant term, which is close to the zero-$T$ susceptibility of the non-critical $\alpha$-YbAlB$_4$, may originate from constant Van Vleck contribution to susceptibility and/or from Pauli susceptibility of the non-critical parts of the Fermi surfaces. Figure 1b shows $M_c/B^{0.5}$ vs. $T/B$ for $\beta$-YbAlB$_4$ in the region shown in the inset ($T \lesssim 3$ K and $B \lesssim 2$ T). Interestingly, the data collapse on a single curve at $T > B$, indicating a scaling relation $M_c = M - M_0 = B^{0.5}\psi(T/B)$. Under fields $B > 0.5$ T, $M_c$ does not follow a single scaling curve. This may be due to a small field-nonlinear contribution to $M_0$, which is neglected in the above analysis. In fact, the scaling relation free
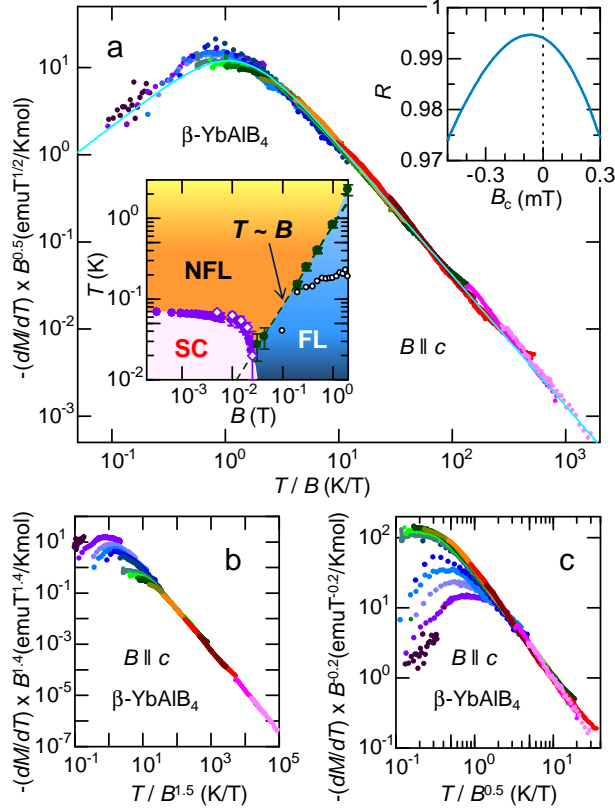
**Figure 2.** (a) Scaling observed for the $c$-axis magnetization at $T \lesssim 3$ K and $B \lesssim 2$ T. The light blue line represents a fit to the empirical Eq. 1 with scaling function $\phi(x) = \Lambda x(A + x^2)^{-n}$ (see text). The right inset shows Pearson's correlation coefficient $R$ for the fit with finite $B_c$ (see text). The best fit is obtained with $\alpha = 1.50 \pm 0.02$ and $B_c = -0.1 \pm 0.1$ mT (see text). The left inset shows the $B$-$T$ phase diagram of $\beta$-YbAlB$_4$ in the low $T$ and $B$ region. The filled circles are the peak temperatures of $-dM/dT$ below which the FL ground state is stabilized. The open circles are the temperature scale $T_{FL}$ below which the $T^2$ dependence of the resistivity is observed [4]. The upper critical fields of the superconducting (SC) phase is shown by open circles and triangles [5, 8]. (b)(c) Scaling plots with $(\delta, \eta) = (1.5, 1.4)$, $(0.5, -0.2)$ for the same data set in Fig. 2a, respectively (see texts).

from this contribution which is obtained after $T$-derivative of the both parts:

$$-\frac{\mathrm{d}M}{\mathrm{d}T} = B^{\alpha-2}\phi\left(\frac{T}{B}\right), \tag{1}$$

with $\alpha = 3/2$ works much better as shown in Figure 2a. Here we can observe a clear power law behavior of $-dM/dT \propto T^{-1.5}$ at $T > B$ (NFL region), which indicates $M \propto T^{-0.5} + const.$, and $T$-linear behavior of $-dM/dT$ at $T < B$ as expected for a FL. This empirical scaling property implies that close to the QCP, below $T \sim 3$ K and $B \sim 2$ T, $\beta$-YbAlB$_4$ has no intrinsic energy scale, and that furthermore, temperature and field are interchangeable variables, whose ratio $T/B$ determines the physical properties. The peak of the scaling curve is located at $T/B \sim 1$ K/T and indicates that the thermodynamic boundary between the FL and NFL regions is on the $T \sim B$ line, as shown in the phase diagram of the inset of Figure 2a. The QC free energy with the form $F_{QC} = B^{\alpha}f(T/B)$ is obtained by integrating both parts of Eq. (1), where $f$ is a scaling function of the ratio $T/B$ with the limiting behavior: $f(x) \propto x^{\alpha}$ in the NFL regime ($x \gg 1$) and $f(x) \propto const + x^2$ in the FL phase ($x \ll 1$). Indeed, $f(x) \propto (A + x^2)^{\alpha/2}$ fits the observed scaling behavior of $dM/dT$ in Eq. (1) with $\phi(x) = \Lambda x(A + x^2)^{\frac{\alpha}{2}-2}$, as shown by the fit in Figure 2a, achieved with $\alpha = 3/2$.

The $T/B$ scaling implies that the critical field $B_c$ of the quantum phase transition is located just at zero field. A finite value of $B_c$ requires the scaling function $f(x)$ and $\phi(x)$ with a ratio $x = T/|B - B_c|$ rather than $x = T/B$. A bound for $B_c$ can be determined by substitution of $x = T/|B - B_c|$ into Eq. (1) while searching the value of $B_c$ that gives the best fit to the experimental data. The inset of Figure 2a shows the Pearson's correlation coefficient $R$ obtained from this fit, indicating that $B_c$ is optimal at $-0.1 \pm 0.1$ mT. The error-bar is only a few times larger than Earth's magnetic field ($\sim 0.05$ mT), more significantly it is two orders

of magnitude smaller than $\mu_0 H_{c2} = 30$ mT. This result is particularly stunning given the large valence fluctuation scale $T_0 \sim 200$ K. Thus $\beta$-YbAlB$_4$ provides a unique example of essentially *zero-field quantum criticality*.

A possibility of other scaling, such as $T/B^\delta$ scaling ($-\mathrm{d}M/\mathrm{d}T = B^{-\eta}\phi(T/B^\delta)$) with $\delta \neq 1$, is checked by scaling plot, $-(\mathrm{d}M/\mathrm{d}T)B^\eta$ versus $T/B^\delta$. Figures 2b and 2c show the trial scaling plots with $(\delta, \eta) = (1.5, 1.4), (0.5, -0.2)$ for the same data set in Fig. 2a, respectively. Here, $\eta$ was adjusted against fixed $\delta$ so that a scaling was realized at high $T$ parts where the NFL power-law behavior was observed. As is clearly seen from the figures, the data at smaller $T/B^\delta$ do not satisfy the scaling by changing $\delta$ away from 1. It turned out that the data in the widest $T$ and $B$ range are scaled with $\delta = 1.0 \pm 0.1$. The detail will be discussed elsewhere [9].

The observation of zero field quantum criticality in valence fluctuating $\beta$-YbAlB$_4$ has a number of significant implications. Conservatively, one might conclude that $\beta$-YbAlB$_4$ is a system in which a fortuitous combination of structure and chemistry fine-tune the critical field $B_c$ to zero. In such a scenario, the QCP would be characterized by gapless soft modes: spin fluctuations in the case of an antiferromagnetic quantum criticality, or critical charge fluctuations associated with mixed valence. In the former case, the system would be literally at the edge of an antiferromagnetic instability, while the latter case would involve a critical end-point of valence instability line [10, 11]. In both cases, the application of pressure is expected to immediately restore the Fermi liquid ground state.

Set against these two possibilities is our observation of a tiny upper bound (0.2 mT) on the critical magnetic field $B_c$. This critical state is so fragile, that a field not much larger than the Earth's magnetic field drives $\beta$-YbAlB$_4$ back into a Fermi liquid. This suggests the intriguing possibility that $\beta$-YbAlB$_4$ is a *quantum critical phase* of matter, to which infinitesimal magnetic field is a relevant perturbation converting it into a Fermi liquid. In this case, non-Fermi liquid properties will be unaffected by pressure. An example of such a state is a gapless spin liquid, in which the presence of an extensive manifold of gapless spin excitations would lead to a critical phase characterized by power-law correlation functions [12, 13]. It is left for future studies, such as pressure experiments, to discriminate between these two alternative scenarios.

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